

Cavity cooling of a nanomechanical resonator by light scattering

I. Favero^{1,*} and K. Karrai^{1,†}

¹*Center for Nanoscience and Fakultät für Physik, Ludwig-Maximilians-Universität,
Geschwister-Scholl-Platz 1, 80539 München, Germany*

(Dated: 13 October 2007)

Abstract

We present a novel method for opto-mechanical cooling of sub-wavelength sized nanomechanical resonators. Our scheme uses a high finesse Fabry-Perot cavity of small mode volume, within which the nanoresonator is acting as a position-dependant perturbation by scattering. In return, the back-action induced by the cavity affects the nanoresonator dynamics and can cool its fluctuations. We investigate such cavity cooling by scattering for a nanorod structure and predict that ground-state cooling is within reach.

PACS numbers: 07.10.Cm,85.85.+j,42.50.Wk,05.40.Jc

Quantum mechanics describes the behavior of matter on different length scales from quarks to collective macroscopic states referring to supra-conductivity and superfluidity. Still, in order to clarify its transition between microscopic and macroscopic range, several experimental programs aim at observing quantum phenomena at larger scales [1, 2, 3]. To this respect, reaching experimentally the quantum ground state of a macroscopic mechanical resonator is appealing, as it would at the same time allow to study in a quantum regime a system with a macroscopic mass, the paradigm of gravitational interactions [4].

Experiments using state of the art cryogeny and capacitive detection techniques have already approached closely the quantum regime for mechanical oscillators with eigenfrequencies ranging from 10 MHz to 1 GHz range [5, 6]. The recent development of optical cooling techniques, either active [7, 8, 9] or passive [10, 11, 12, 13, 14, 15], creates hope for reaching this regime beyond the possibilities offered by nowadays cryogenic methods. In optical cooling, the low noise photons of a laser source are used to extract thermal energy from the mechanical oscillator and offer at the same time an extremely sensitive tool to read its mechanical fluctuations [16]. The passive cooling technique notably [10, 11, 12, 13, 14, 15], also called self-cooling technique, is analogous to Doppler or cavity cooling of atoms [17, 18, 19, 20, 21] and avoids adding noise in the mechanical system. It is therefore expected to reach in principle physical fundamental limits. This technique relies on the intrinsic back-action of light on mechanical degrees of freedom in opto-mechanical systems where photothermal pressure or radiation pressure effects can arise [22]. It has already been used to cool different kind of mechanical resonators, ranging from millimetric mirrors [11, 12] to AFM microlever mirrors [10], toroid microcavities [13] and wavelength sized micro-mirrors [14]. In all these cases, the mechanical resonator to be cooled must confine the light in an optical cavity and has therefore to be larger than the wavelength of photons. In this paper, we describe a novel method for cooling optically and passively the motion of a nanomechanical resonator smaller than the wavelength of photons. Our proposal, using back action in a situation of intense coupling between optics and nanomechanics, breaks the diffraction barrier limit allowing the investigation of quantum phenomena in nanomechanical systems [23].

The principle for achieving cooling is the following: the nanomechanical resonator fluctuates under Brownian motion with a noise distribution peak at its lowest eigenfrequency f_0 . When placed in the mode of a high finesse Fabry-Perot cavity at resonance with a laser of wavelength λ , it scatters the cavity photons, as depicted in figure 1. Because photons

circulate several times back and forth in the cavity, they need a finite time τ to reach a new equilibrium after each scattering event. This induces a delay in their back action on the motion of the resonator through radiation pressure or dipolar forces. Exploiting this retarded back-action, we show here that an additional optically induced viscous damping is obtained, resulting in a net cooling of the nanoresonator vibrational fluctuations. Photons leaving the cavity carry away the excess energy.

Cavity cooling of atoms was already explored exploiting a detuning of the cavity induced by the dispersive response of an atom moving in the cavity [18, 19, 20]. Aiming here at cooling a solid-state nanomechanical object, we strongly deviate from this atomic case. Loss mechanisms typical for solid-state systems, such as exciton or polariton absorption resonances, interband and intraband absorption, or even Rayleigh scattering out of the cavity call for a novel approach including both the absorptive and dispersive optical response of the nanomechanical scatterer placed in the cavity. The problem of a scattering element placed in a focused gaussian beam in the paraxial limit has been shown to be nearly equivalent to that of a thin plane of conductivity σ at optical frequencies placed in a plane wave [24, 25, 26]. Therefore we model the nanoscatterer as a thin plane of transmittance $1/(1+\Sigma)$ and reflectance $-\Sigma/(1+\Sigma)$ where Σ consists of a real and of an imaginary part $\Sigma = \Sigma_1 + i\Sigma_2$ and where $\Sigma = \sigma/2\epsilon_0 c$. The prescription to determine Σ_1 and Σ_2 relies on a simple measurement of the reflectance and the transmittance of the nanoscatterer in a focused gaussian beam of size matching the cavity mode. The field amplitude distribution in the cavity perturbed by the nanoscatterer are then computed using a system of three transfer matrixes for plane waves [24].

In an empty Fabry-Perot cavity with mirrors of high reflectance r , a laser coupled resonantly creates a steady-state intensity distribution at position x_0 along the optical axis that can be approximated with $P(x_0) = 2gP_0\sin^2(kx_0)$ (figure 1), where P_0 is the optical power impinging on the cavity, $k = (2\pi/\lambda)$ is the wave number and g the cavity amplification factor, which relates to the cavity finesse f such $g = (2f)/\pi \simeq 2/(1 - |r|^2)$. The nanoresonator is first placed at position x_0 along this intensity distribution. The cavity is then tuned to its maximum of transmission by adjusting the back mirror position, leading to a transmission $T(x_0) = 1/[1 + \Sigma_1(1 + 2g\sin^2(kx_0))]^2$. The cavity is thereafter actively stabilized on this resonance to enhance the interaction of photons with the nanoresonator, as well as to reduce the contribution of the laser noise on the noise of the transmitted photons. At finite tem-

perature, the scatterer fluctuates around x_0 with a small amplitude x such $kx \ll 1$, leading to a fluctuation of the transmission $T(x_0)$ which can be written after a straightforward but cumbersome calculation:

$$T(x_0, x) = 1/[H_1(x_0, x) + H_2(x_0, x)] \quad (1)$$

with

$$H_1(x_0, x) = [1 + \Sigma_1(1 + 2g \sin^2(kx_0)) + 2g\Sigma_1 \sin(2kx_0)kx]^2 \quad (2)$$

and $H_2(x_0, x) = 4g^2\Sigma_2^2 \sin^2(kx_0)(k^2x^2)$. The transmission fluctuation enables read-out of the nanomechanical resonator motion linear in x , provided that Σ_1 is non-zero. The purely dispersive effects relating to Σ_2 contribute only to a non-linear response in x^2 . Loss mechanisms, for instance escape of photons out of the cavity or absorption by the nanoscatterer, are therefore mandatory for an efficient read-out of the nanomechanical motion fluctuation. The location along axis x_0 of maximum sensitivity for this read-out (i.e. the extrema of dT/dx) depends on the factor $g\Sigma_1$, which is the ratio between the nanoresonator-induced losses and the intrinsic losses of the cavity.

The essence of passive cooling relies on a delayed response of the back-action force F acting on the mechanical system, here the nanoresonator. The effective temperature T_{eff} reached by cooling can be written $T_{eff} = T_b(\Gamma/\Gamma_{eff})$ where Γ_{eff} is the optically modified damping rate of the nanomechanical resonator, Γ being its natural damping and T_b the bath temperature. Γ_{eff} is in a classical limit given by [10] $\Gamma_{eff} = \Gamma[1 + Q_m(\omega_0\tau/(1+\omega_0^2\tau^2))\nabla F/K]$, where the gradient of the force upon nanoresonator motion $dF(x_0, x)/dx$ is noted ∇F , $Q_m = \omega_0/\Gamma$ is the mechanical quality factor of the nanoresonator, $\omega_0 = 2\pi f_0$ its eigenfrequency, K its spring constant and τ the delay time of the force. In an empty cavity at resonance with transmission T_0 , the delay time of photon pressure on the mirrors is given by the empty cavity storage time $\tau_c = g\tau_0$ where τ_0 is the time of flight of photons through the cavity. When the nanoresonator is placed at position x_0 in the cavity and the cavity hold on resonance with an average transmission $T(x_0)$, the storage time is modified to $\tau = \tau_c\sqrt{T(x_0)/T_0}$. We will study cases where the variation of the storage time over the nanoresonator position fluctuations is negligible.

The optical force F_0 acting on the nanoresonator when placed at the waist of a forward propagating gaussian wave of power P_0 matched to the geometry of the cavity mode can be obtained calculating the transmission and reflection by the nanoscatterer and relating it to a

net momentum transfer $F_0 = (2\Sigma_1 + 2\Sigma_2^2 - 2\Sigma_1^2)P_0/c$. At first order, this force is proportional to Σ_1 and relies then on loss mechanisms. In our cavity scheme, the nanoscatterer is placed at x_0 and the cavity maintained at resonance. In this configuration, a calculation of the energy-flow imbalance between the waves travelling forward and backward on both sides of the fluctuating nanoresonator leads to the static photon-induced force $F(x_0, x)$ acting on it: $F = (P_0/c)[G_1 + G_2]T$ where $G_1 = 2\Sigma_1[2g\Sigma_1 \sin^2(k(x_0 + x)) + \Sigma_1\sqrt{2g} \sin(k(x_0 + x)) \cos(k(x_0 + x)) + (1 + \Sigma_1)]$ and $G_2 = 4g\Sigma_2^2 \sin^2(k(x_0 + x)) - 2g\Sigma_2 \sin(2k(x_0 + x)) - 2\Sigma_2\sqrt{2g} \cos(2k(x_0 + x)) + 2\sqrt{2g}\Sigma_2^2 \sin(2k(x_0 + x)) + 2\Sigma_2^2$. We will focus on situations where scattering of photons by the nanoresonator is small ($\Sigma_1, \Sigma_2 \ll 1$) and the finesse of the cavity large $g \gg 1$.

We first study the case where dispersive scattering is very weak ($\Sigma_2 \simeq 0$) and losses induced by the presence of the nanoresonator are dominating other optical losses in the cavity $g\Sigma_1 \gg 1$. Figure 2 displays the case $g\Sigma_1 = 10$, with $g = 2 \times 10^4$ and $\Sigma_1 = 5 \times 10^{-4}$. In this so-called lossy limit, we see in figure 2 that the force F always points along the incoming photons, which is a reminder of the broken symmetry of the system induced by the presence of the laser source on the left hand-side of the cavity. The amplitude of the force is at most equal to F_0 , the force without cavity. The cavity does not amplify the force but provides it with a gradient over x and a retardation. As a first numerical illustration, we consider the case of a cylindrical single wall carbon nanotube of radius $r = 0.8$ nm and length $l = 5$ μ m oscillating at $f_0 = 205$ KHz according to the formula $f_0 = 0.281(r/l^2)\sqrt{E/\rho}$ [27] (Young modulus $E = 1$ TPa and density $\rho = 1.925$ g/cm³ for a single wall nanotube) with a spring constant $K = 7.7 \times 10^{-9}$ N/m according to $K = (3\pi/4)Er^4/l^3$, a mechanical quality factor $Q_m = 10^3$ [28] and $\Sigma_1 = 5 \times 10^{-4}$ placed in a cavity of $g = 2 \times 10^4$ [29] at position of maximum (dF/dx) as indicated in figure 2. For a cavity of 50 μ m length illuminated with $P_0 = 1$ mW of laser power at $\lambda = 780$ nm, this would lead to $\Gamma_{eff} \simeq \Gamma(1 \pm 47)$. When $\Gamma_{eff} < 0$ [30], a regime of mechanical self oscillation is reached and governed by a purely lossy mechanism in the radiation pressure, in contrast to recently developed optical back-action schemes for mechanical resonators [10, 11, 12, 13, 14, 15]. In a second numerical illustration, we consider a cylindrical diamond nanorod ($E = 1.14$ TPa and $\rho = 3.52$ g/cm³) of radius $r = 4$ nm, length $l = 0.5$ μ m, $K = 5 \times 10^{-3}$ N/m oscillating at 81 MHz with $Q_m = 10^4$ [31]. The cavity-induced modification of the damping becomes $\Gamma_{eff}/\Gamma \simeq (1 \pm 7 \times 10^{-2})$, namely negligible. Still, the interest of using such a stiff nanostructure will appear in the following section.

We now study the contribution of dispersive effects related to Σ_2 . Figure 3 displays the optical force F acting on the nanoresonator when $g = 2 \times 10^4$ and $\Sigma_2 = 5 \times 10^{-4}$. Using a non-vanishing $\Sigma_1 = 10^{-5} \ll \Sigma_2$ allows for a read-out of x in the transmission. The force can now be made positive or negative, mimicking dipolar forces. Its direction depends on the energy transfer between the two optical resonators formed on the left hand-side of the cavity by the input mirror and the nanoscatterer and on the right hand-side by the nanoscatterer and the back mirror. Interestingly enough and in contrast to the purely lossy case, F is here amplified by the cavity. The cavity role is to provide as before a force gradient, a retardation, but on top of that an amplification of the force. This three-fold advantage is apparent in the following numerical illustration. We consider the same carbon nanotube as previously and we find that the maximum damping amplification factor becomes $\Gamma_{eff}/\Gamma = 4.7 \times 10^5$, four orders of magnitude larger than in the lossy case. At the same time, an optical spring effect occurs, modifying the effective spring constant to $K_{eff} = K[1 - (1/(1 + \omega_0^2\tau^2))\nabla F/K]$ and preventing efficient cooling by driving the nanotube into an instability regime already at moderate power [32]. The cooling is practically limited to a temperature $T_{eff}/T = 0.2$. In contrast, the diamond nanorod is stiff enough to preclude early appearance of instabilities, allowing to reach under the same experimental conditions a temperature reduction factor of 700. Starting from a standard liquid helium bath environment at 1.8 K, the cooling mechanism would bring the motion of the diamond nanorod to an effective temperature of 2.5 mK, at which the quantum limit is reached.

The study of small size nanoresonators, like carbon nanotubes or diamond nanorods, appears then extremely appealing for investigation of optomechanical phenomena at the nanoscale. However we should note that reducing the size of the nanoresonator may also lead to a reduction of $|\Sigma|$ and hence of the force F . The advantage of using a solid state system is then apparent since it allows a wide variation of Σ_1 and Σ_2 , by choosing the photons wavelength or the material of the nanoscatterer, and hence a direct engineering of the cooling efficiency. Using an excitonic resonance in a carbon nanotube [33] or an implanted nitrogen-vacancy impurity resonance in diamond [34] for instance would allow to amplify both the absorptive and the dispersive response, with a controllable ratio between the two. The high aspect ratio of nanorods offers additionally the possibility of a selective coupling to distinct polarisation modes of the cavity and hence a supplementary degree of freedom in the optimization of cooling. Together with the reduction of the number of phonon channels

eventually coupling the nanoresonator to the thermal bath, it makes nanorod structures very promising candidates to study mechanical quantum phenomena at the nano-microscale.

In conclusion, we have studied a cavity cooling scheme of a nanomechanical resonator scattering photons within a high finesse Fabry-Perot cavity. This geometry, on top of allowing using sub-wavelength sized objects, offers the possibility to engineer separately ultra high finesse cavities on the one hand and high quality factor mechanical resonators on the other hand, an issue usually limiting opto-mechanical cooling experiments. Cavity cooling in a purely dispersive limit and in a detuned cavity was discussed for atoms and molecules [18, 19, 20] and very recently for a macroscopic membrane [35, 36]. Our scheme relies here on a tuned cavity and in contrast to molecules or atoms, the nanoresonator is attached to a holder and cannot escape out of the cavity, offering the possibility to set the motion of the nanoresonator into self-oscillation [30]. The onset of non linear opto-mechanical behaviors in this new situation of intense coupling between nanomechanics and optics will require a more complete theory, as developed in ref [37] to study multi-stabilities in Fabry-Perot cavities. Optical control of vibration properties of nanomechanical systems would open new routes, not only for sensing applications, but also for testing the quantum mechanical description of tiny objects.

We thank C. Metzger, J. P. Kotthaus, J. Reichel, T. W. Hänsch, F. Marquardt, T. Kippenberg and B. Lorenz for stimulating discussions. I. Favero acknowledges support of the Alexander Von Humboldt foundation.

* Electronic address: ivan.favero@physik.uni-muenchen.de

† Electronic address: karrai@lmu.de

- [1] A. J. Leggett, J. Phys. Condens. Matter, **14**, R415-R451 (2002).
- [2] S. Haroche, Physics today, 36 (1998).
- [3] W. Marshall, C. Simon, R. Penrose, and D. Bouwmeester, Phys. Rev. Lett. **91**, 130401 (2003).
- [4] R. Penrose, in Mathematical Physics 2000 edited by A. Fokas, A. Grigoryan, T. Kibble, and B. Zegarlinski (Imperial College, London).
- [5] M. D. LaHaye, O. Buu, B. Camarota, and K. C. Schwab, Science **304**, 74 (2004).
- [6] R. G. Knobel, A. N. Cleland, Nature **424**, 291 (2003).

- [7] S. Mancini, D. Vitali, and P. Tombesi, Phys. Rev. Lett. **80**, 688 (1998).
- [8] P. F. Cohadon, A. Heidmann, and M. Pinard, Phys. Rev. Lett. **83**, 3174 (1999).
- [9] D. Kleckner and D. Bouwmeester, Nature **444**, 75 (2006).
- [10] C. Hühberger Metzger and K. Karrai, Nature **432**, 1002 (2004).
- [11] O. Arcizet, P. F. Cohadon, T. Briant, M. Pinard and A. Heidmann, Nature **444**, 71 (2006).
- [12] S. Gigan, H. R. Böhm, M. Paternostro, F. Blaser, G. Langer, J. B. Hertzberg, K. Schwab, D. Bäuerle, M. Aspelmeyer and A. Zeilinger, Nature **444**, 67 (2006).
- [13] A. Schliesser, P. Del’Haye, N. Nooshi, K. J. Vahala, and T. J. Kippenberg, Phys. Rev. Lett. **97**, 243905 (2006).
- [14] I. Favero, C. Metzger, S. Camerer, D. König, H. Lorenz, J. P. Kotthaus, and K. Karrai, Appl. Phys. Lett. **90**, 104101 (2007).
- [15] T. Corbitt, Y. Chen, E. Innerhofer, H. Müller-Ebhardt, D. Ottaway, H. Rehbein, D. Sigg, S. Whitcomb, C. Wipf, and N. Mavalvala, Phys. Rev. Lett. **98**, 150802 (2007).
- [16] O. Arcizet, P. F. Cohadon, T. Briant, M. Pinard, A. Heidmann, J.-M. Mackowski, C. Michel, L. Pinard, O. Francais, and L. Rousseau, Phys. Rev. Lett. **97**, 133601 (2006).
- [17] T. W. Hänsch, and A.L. Schawlow, Optics Commun. **13**, 68 (1975).
- [18] P. Horak, G. Hechenblaikner, K. M. Gheri, H. Stecher, and H. Ritsch , Phys. Rev. Lett. **79**, 4974 (1997).
- [19] V. Vuletic and S. Chu , Phys. Rev. Lett. **84**, 3787 (2000).
- [20] P. Maunz, T. Puppe, I. Schuster, N. Syassen, P. W. H. Pinkse, and G. Rempe, Nature **428**, 50 (2004).
- [21] K. Karrai, I. Favero, and C. Metzger, arXiv **0706.2841**(2007).
- [22] V. B. Braginsky and A. B. Manukin, *Measurements of weak forces in Physics experiments* (Chicago University Press, Chicago,1977).
- [23] I. Wilson-Rae, P. Zoller, and A. Imamoglu, Phys. Rev. Lett. **92**, 075507 (2004).
- [24] K. Karrai, and R. J. Warburton, Superlattices and Microstructures **33**, 311 (2003).
- [25] A. Högele, PhD Thesis LMU (2006).
- [26] S. J. van Enk, and H. J. Kimble, Phys. Rev. A **63**, 023809 (2001).
- [27] W. Weaver, S. P. Timoshenko, and D. H. Young, *Vibration problems in engineering*, John Wiley and Sons, (1990).
- [28] B. Witkamp, M.Poot, and H. S. J. van der Zant, Nano Letters **6**, 12, 2904 (2006).

- [29] Y. Colombe, T. Steinmetz, G. Dubois, F. Linke, D. Hunger, and J. Reichel, *Nature* **450**, 272 (2007). We are currently using a similar type of cavity. I. Favero, D. Hunger, S. Stapfner, J. Reichel, E. Weig, H. Lorenz, and K. Karrai, unpublished.
- [30] C. Höhberger Metzger and K. Karrai, *Nanotechnology*, 4th IEEE Conference on Nanotechnology, 419-421 (2004).
- [31] J.W. Baldwin, M.K. Zalalutdinov, T. Feygelson, B.B. Pate, J.E. Butler, B.H. Houston, *Diamond and related materials*, **15**, 2061 (2006).
- [32] C. Metzger, I. Favero, A. Ortlieb, and K. Karrai, arXiv **0707.4153**.
- [33] F. Wang, G. Dukovic, L. E. Brus, and T. F. Heinz, *Science* **308**, 838 (2005).
- [34] J. Meijer, B. Burchard, M. Domhan, C. Wittmann, T. Gaebel, I. Popa, F. Jelezko, and J. Wrachtrup, *Appl. Phys. Lett.* **87**, 261909 (2005).
- [35] J. D. Thompson, B. M. Zwickl, A. M. Jayich, F. Marquardt, S. M. Girvin, and J. G. E. Harris, arXiv **0707.1724**.
- [36] M. Bhattacharya, H. Uys, and P. Meystre, arXiv **0708.4078**.
- [37] F. Marquardt, J. G. E. Harris, S. M. Girvin, *Phys. Rev. Lett.* **96**, 103901 (2006).

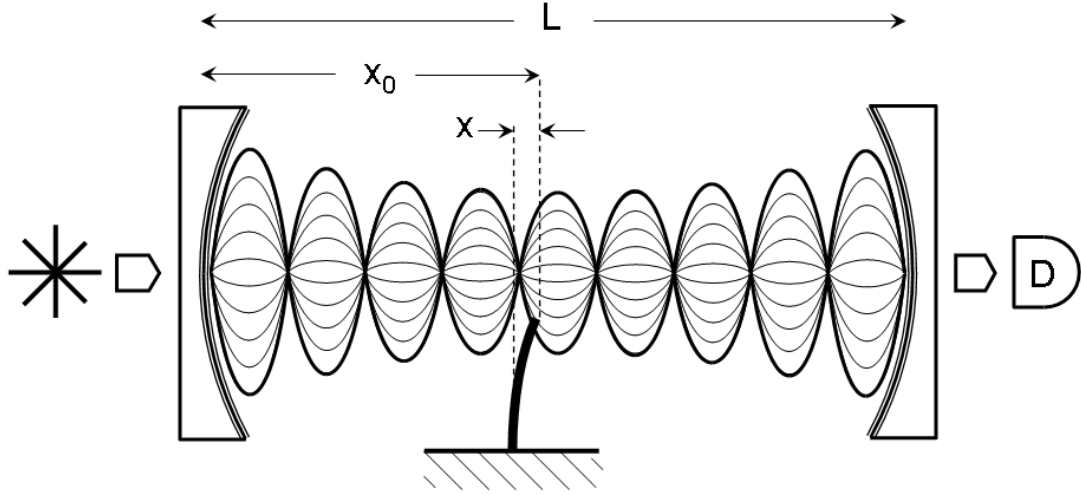


FIG. 1: A nanomechanical resonator scattering photons in a high finesse Fabry-Perot cavity resonantly coupled to a laser on its left-hand side.

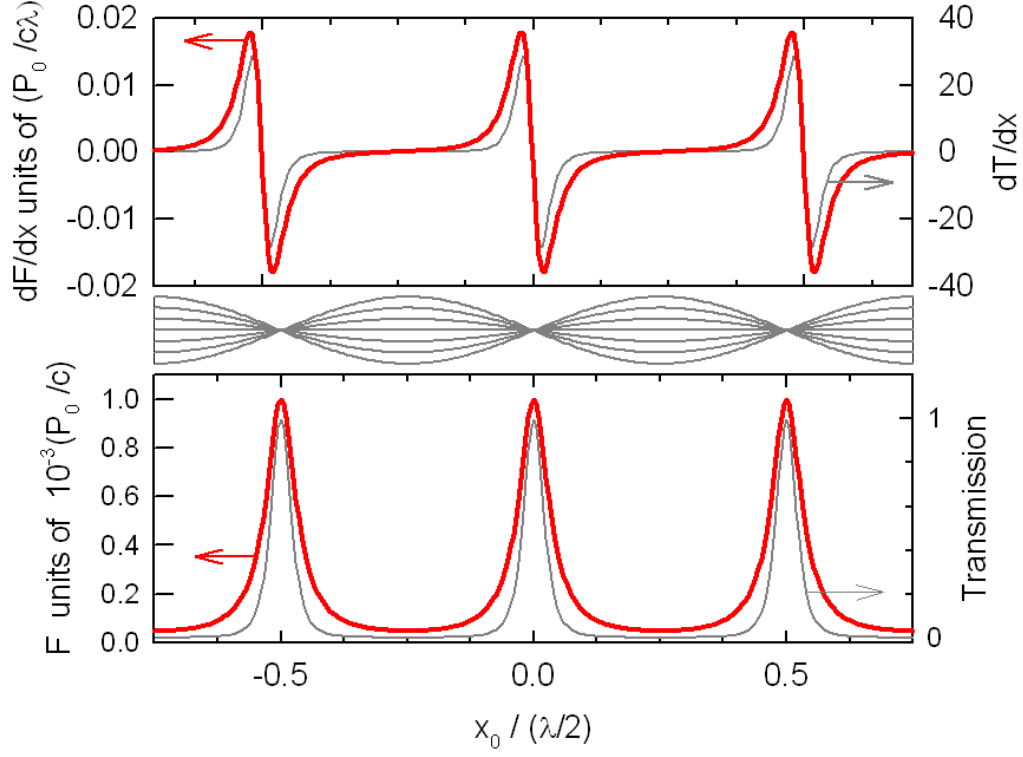


FIG. 2: Case: $g\Sigma_1 = 10$, with $g = 2 \times 10^4$ and $\Sigma_1 = 5 \times 10^{-4}$. Bottom: Transmission of the cavity $T(x_0)$ and optical force F for zero motion fluctuation of the nanoresonator ($x = 0$). Middle: amplitude snap-shots of the standing wave in the corresponding empty cavity. Top: Gradient of the force and of the transmission upon the nanoresonator fluctuation x , as a function of its average position x_0 .

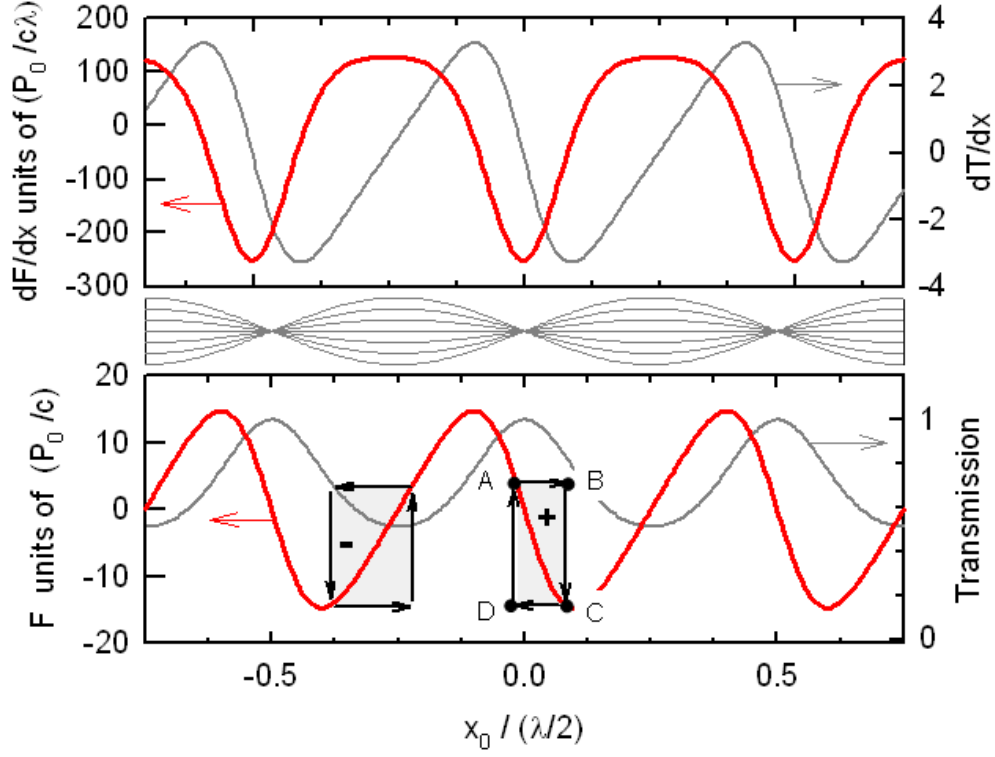


FIG. 3: Case: $g = 2 \times 10^4$, $\Sigma_1 = 10^{-5}$ and $\Sigma_2 = 5 \times 10^{-4}$. The optical modification of the nanoresonator damping can be described in a thermodynamical manner following the cycles on the lower part of the figure. The nanoscatterer moves suddenly from point A to point B, leaving no time to the intensity distribution in the cavity and hence the force to follow. Waiting long enough from B to C allows the force to slowly recover its equilibrium level. Closing back the cycle leads to a hysteresis which is characteristic of an irreversible energy transfer and produces a net viscous force on the nanoresonator, which can damp or amplify the motion [10, 22].